See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/322675442

X-ray Environments of Supermassive Black Holes

Thesis · May 2017

DOI: 10.13140/RG.2.2.26190.97606

citations 0		READS 144			
1 author					
	Alankar Dutta Indian Institute of Science 2 PUBLICATIONS 0 CITATIONS SEE PROFILE				
Some of	Some of the authors of this publication are also working on these related projects:				



Cosmological Simulation study View project

Astrophysics and Cosmology View project

PRESIDENCY UNIVERSITY

BACHELOR THESIS

X-ray Environments of Supermassive Black Holes

Author: Alankar DUTTA

Advisor: Prof. Suchetana CHATTERJEE

A thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science

in the

Presidency University Department of Physics



Declaration of Authorship

I, Alankar DUTTA, declare that this thesis titled, "X-ray Environments of Supermassive Black Holes" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a bachelor degree at this University.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date:

"Black holes are where God divided by zero."

Albert Einstein

Presidency University

Department of Physics

Abstract

X-ray Environments of Supermassive Black Holes

by Alankar DUTTA

In this work I have studied the X-ray Environments of supermassive black holes from The MassiveBlack-II Simulation (Khandai et al 2015¹) of GADGET 2 (Cosmological N-body/SPH simulator; Springel V., 2005²). I have simulated maps of Xray emission from the distribution of gas around some of the most massive black holes from the simulation at redshift z = 1. For this, I have assumed a model of Bremsstrahlung emission from the gas around the blackholes which has a typical energy of around 1-10 keV. I have studied the scaling relation of Luminosity L (hence Accretion rates \dot{M} as $L \propto \dot{M}$) and black hole mass M_{BH} ($\dot{M} \propto M_{BH}^2$ Bondi 1952,³ Bondi & Hoyle 1944,⁴ Hoyle & Lyttleton 1939⁵) of these blackholes and found deviation from the Bondi Scaling Relation.⁶ I have made references to and discussed about a similar scaling relation found by Chatterjee et al (2008).⁷ They did a simulation study using the DiMatteo et al (2008)⁸ GADGET 2 simulation (See Appendix B) and studied the Sunyaev-Zeldovich effect (Sunyaev & Zeldovich 1972⁹). They found a scaling relation between accretion rate and black hole mass. My results are found to be broadly in agreement with them. I have also discussed some plausible physical phenomena that might be resposible for this departure from Bondi Relation. I plan to investigate it further and make comparisons of my simulations with observations in the future.

⁶Bondi and Hoyle, "On the mechanism of accretion by stars", op. cit.

¹Nishikanta Khandai et al. "The MassiveBlack-II simulation: the evolution of haloes and galaxies to z 0". In: *Monthly Notices of the Royal Astronomical Society* 450.2 (2015), pp. 1349–1374.

²Volker Springel. "The cosmological simulation code GADGET-2". In: *Monthly notices of the royal astronomical society* 364.4 (2005), pp. 1105–1134.

³HJ Bondi. "On spherically symmetrical accretion". In: *Monthly Notices of the Royal Astronomical Society* 112.2 (1952), pp. 195–204.

⁴Hermann Bondi and Fred Hoyle. "On the mechanism of accretion by stars". In: *Monthly Notices of the Royal Astronomical Society* 104.5 (1944), pp. 273–282.

⁵F. Hoyle and R. A. Lyttleton. "The effect of interstellar matter on climatic variation". In: *Proceedings* of the Cambridge Philosophical Society 35 (1939), p. 405. DOI: 10.1017/S0305004100021150.

⁷Suchetana Chatterjee et al. "Simulations of the Sunyaev–Zel'dovich effect from quasars". In: *Monthly Notices of the Royal Astronomical Society* 390.2 (2008), pp. 535–544.

⁸T. Di Matteo et al. "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies". In: *The Astrophysical Journal* 676, 33-53 (Mar. 2008), pp. 33–53. DOI: 10.1086/524921. arXiv: 0705.2269.

⁹R. A. Sunyaev and Y. B. Zeldovich. "The Observations of Relic Radiation as a Test of the Nature of X-Ray Radiation from the Clusters of Galaxies". In: *Comments on Astrophysics and Space Physics* 4 (Nov. 1972), p. 173.

I am especially grateful to Prof. Suchetana Chatterjee for the countless hours she has spent with me discussing about my work and also all the personal difficulties I faced. She tirelessly provided all the guidance that was so vital in my life. She has been an example of energy and optimism to me and the group meetings she arranged played a monumental role in learning different the concepts and techniques which I implemented in my work. Also, my colleagues especially Ms. Rudrani Kar Chowdhuri, Ms. Shreya Kumbhakar and Mr. Anwesh Majumdar were extremely kind and supportive and helped me in different occassions while doing this project. Special thanks to the SciPy conferences that enabled me to write efficient fast running codes.

Contents

De	eclara	tion of	Authorship	iii	
Ał	ostrac	t		\mathbf{v}	
Ac	knov	vledgen	nents	vi	
1	Cosmological Simulations: Creating Universe to underderstand our Uni-				
	vers	e Technoda	und on	1	
	1.1	Introd	lection	1	
	1.2		Importance of Cosmological Simulations	1	
		1.2.1	Current Trando in Cosmological Simulations	1	
		1.2.2	Different Approaches to the Simulation	2	
		1.2.3	Different Approaches to the Simulation	2	
			Troo (Gravity Only) Method	2	
			Particle (Gravity Only) Method	2	
			Farticle (Gravity Only) Mesh $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	5	
			Introducing Hydrodynamics into Simulation	5	
			Adding Complexities	6	
		1 2 4	Initial Conditions	07	
		1.2.4	Comparison between AMP and CPH codes	7	
		1.2.5	Examples of Cosmological Simulators	2	
		1.2.0		0	
2	Env	ironme	nts of Supermassive Black holes	10	
	2.1	Introdu	uction	10	
	2.2	SMBH	Environments	10	
		2.2.1	Effect of SMBH on its environment	11	
		2.2.2	Seeds for the growth of supermassive Black holes	12	
	2.3	A stud	y of Supermassive Black hole environments using MassiveBlack-		
		II GAE	OGET Simulation	13	
		2.3.1	About The MassiveBlack-II Simulation	13	
		2.3.2	Working with MB II output at $z \approx 1$	15	
		2.3.3	Simulation of X-ray Map around BlackHoles	15	
		2.3.4	Scaling Relations	19	
	2.4	Conclu	ision	24	
Α	Cos	nologia	cal Parameters	26	
	A.1	Planck	Collaboration Cosmological parameters 2015	26	
В	Di N	/latteo e	et al. 2008 GADGET 2 Cosmological Simulation	27	
-	B.1	Param	eters of the Di Matteo et al. 2008 simulation	27	
Bibliography 28			28		

vii

List of Figures

1.1 1.2 1.3	Tree Method 3 AMR Method 4 ENZO and GADGET code Comparison 8	3 4 8
2.1	Seed Black holes	2
2.2	Simulated X-ray Map Projection	7
2.3	Simulated X-ray Map Projection	8
2.4	Mass-Luminosity Scatter Plot	9
2.5	Mass-Luminosity Histogram 20	C
2.6	Mass-Luminosity Scaling Relation of SMBH	1
2.7	Mass-Luminosity Spearmann Rank ρ	2
2.8	Spearmann p value for Mass-Luminosity relation	3
2.9	Simulated X-ray Map	5
2.10	Simulated SZ Map 25	5
A.1	Cosmological Parameters	6

List of Tables

2.1	Basic simulation parameters of MB II	13
2.2	Mass-Luminosity Scaling	24
B .1	Di Matteo et al. 2008 simulation	27

List of Abbreviations

GADGET	GAlaxies with Dark matter and Gas intEracT
СМВ	Cosmic Microwave Background
SMBH	Super Massive BlackHole
FRW	Friedmann Robertson Walker
SDSS	Solan Digital Sky Survey
AGN	Active Galactic Nuclei
LIGO	Laser Interferometer Gravitational-Wave Observatory
SZ	Sunyaev-Zeldovich Effect
MB-II	MassiveBlack II Simulaion
FOF	Friends-Of-Friends
ISM	InterStellar Medium
MPI	Message Passing Interface
GPU	Graphics Processing Unit
PSF	Point Spread Function

Dedicated to my mom and Shreya, The binary star in my sky...

Chapter 1

Cosmological Simulations: Creating Universe to underderstand our Universe

1.1 Introduction

In this chapter, I will outline the motivation behind my work by first, giving a very brief account about the various computational methods used in N-body Cosmological simulations and their importance in my work. Discussions related to the simulations I have worked with and every other details specific to my work are discussed later in Chapter 2.

1.2 Cosmological Simulations

Modern cosmological observations allow us to study in great detail the evolution and history of the large scale structures of our universe and their hierarchy. The fundamental problem of obtaining tight constraints on the cosmological parameters (See Appendix A), and hence a precise given cosmological model, requires very accurate modelling of the observed structures. I will review one such effective technique of studying structure formation namely through simulations, emphasising both their advantages and shortcomings.

1.2.1 Importance of Cosmological Simulations

In the hierarchical picture of structure formation, small objects collapse first and then undergo merger to form larger and larger structures in an extremely complex manner. This formation process reflects on the intricate structure of galaxies and clusters and their properties depend on how the thousands of smaller objects in the cluster are destroyed or survive within the gravitational potential. In order to model these processes realistically, one needs to resort to numerical simulations which are capable of resolving and following correctly the highly complex and intricate non-linear dynamics involved in these scenarios. The first generation of cosmological simulations employed N-body dynamics to study the cluster formation and evolution of dissipationless dark matter component. This provided evidences that the the dark matter doesn't consist of massive neutrinos they might be made up of cold collision-less particleswhich reproduces the current observed clustering.¹

¹Simon DM White, CS Frenk, and Marc Davis. "Clustering in a neutrino-dominated universe". In: *The Astrophysical Journal* 274 (1983), pp. L1–L5.

1.2.2 Current Trends in Cosmological Simulations

Usually, choosing the simulation setup is a trade-off between the size of the region that one has to simulate to fairly represent the objects of interest, and the resolution needed to resolve the objects at the required level of detail. Typical sizes of the simulated volume are a megaparsec scale for an individual galaxy, tens to hundreds of megaparsecs for a galaxy population, and several hundreds of Mpc for a galaxy cluster population (de Vaucouleurs 1971).² The mass resolution varies from about $10^5 M_{\odot}$ to $10^{10} M_{\odot}$. Nowadays, one can typically attain a resolution of a few hundreds of parsec so that individual galaxies can be resolved in the simulation box (Dolag et al 2008).³

1.2.3 Different Approaches to the Simulation

Direct (Gravity Only) Method

Over most of the cosmic time of interest as far as structure formation is concerned, the Universe is dominated by dark matter (Ryden 2002).⁴ The most favourable model turned out to be the so-called cold dark matter (CDM) model. The CDM can be modelled as a collisionless, non-relativistic fluid particles in an expanding background universe described by the FRW metric.

The most direct way to solve the N-body problem is to sum directly the contributions of all the individual particles to the total gravitational potential

$$\phi(\vec{r}) = -G \sum_{j=1}^{N} \frac{m_j}{(|\vec{r} - \vec{r_j}|^2 + \epsilon^2)^{\frac{1}{2}}}$$

This sum would represent the potential which generates the particles' acceleration. The particles do not represent elementary dark matter particles (this is true for any other particles in a simulation), statistical mass distribution, and only their collective properties are of interest. Close encounters between individual particles are irrelevant to the physical problem under consideration. What is physically important is the gravitational force between two particles smoothed by introducing the gravitational softening ϵ . This direct-sum approach is considered to be the most accurate technique, and is used for problems where very high precision is needed. However this method has the disadvantage of being extremely computation intensive for even a considerably small number of particles. This is because the computing time is $\propto N^2$ where N is the number of particles.

Tree (Gravity Only) Method

It is a multipole expansion algorithm applied in a hierarchical manner. In this method distant particles are grouped into larger cells, and their gravity is calculated by means of a single multipole force. Instead of requiring N - 1 force evaluations per

²G. de Vaucouleurs. "The Large-Scale Distribution of Galaxies and Clusters of Galaxies". In: *Publications of the Astronomical Society of the Pacific* 83.492 (1971), p. 113. URL: http://stacks.iop.org/1538-3873/83/i=492/a=113.

³K. Dolag et al. "Simulation Techniques for Cosmological Simulations". In: *Space Science Reviews* 134.1 (2008), pp. 229–268. ISSN: 1572-9672. DOI: 10.1007/s11214-008-9316-5. URL: http://dx.doi.org/10.1007/s11214-008-9316-5.

⁴Barbara Sue Ryden. *Introduction to cosmology*. Includes bibliographical references (pages [235]-237) and index. San Francisco: Addison-Wesley, [2003] ©2003, [2003]. URL: https://search.library.wisc.edu/catalog/999940494802121.

particle, as needed in a direct-summation approach, the gravitational force on a single particle can be computed with substantially fewer operations. In this method the sum usually reduces to Nlog(N) operations. Starting at the root node, a decision is made as to whether or not the multipole expansion of the node provides an accurate enough force as is necessary in a simulation. If the answer is 'yes', the multipole force is evaluated and if the answer is 'no', the node is "opened", i.e. it is divided into smaller daughter nodes (down the tree branch) are considered in turn (Barnes & Hut 1986)⁵(See Figure 1.1).



FIGURE 1.1: Schematic illustration of the Barnes & Hut (1986) oct-tree in two dimensions. The particles are first enclosed in a square (root node). This square is then iteratively subdivided into four squares of half the size, until exactly one particle is left in each final square (leaves of the tree). In the resulting tree structure, each square can be the progenitor of up to four siblings. Taken from Springel et al. (2001b).

Particle (Gravity Only) Mesh

The Particle-Mesh (PM) method treats the force as a field quantity by computing it on a mesh. Differential operators are replaced by finite difference approximations. Potentials and forces at particle positions are obtained by interpolation from the mesh-defined values. First, the density on the mesh-points is computed by interpolating densities to the mesh from the particle positions. Second, the density field is transformed to Fourier space, where the Poisson equation is solved to obtain the mesh-defined potentials. In a third step, the forces for the individual particles are obtained by another interpolation of the derivatives of the potentials to the particle positions from the mesh-grid points. The shortcoming of this method; namely its limited resolution and dependence on the mesh-grid rather than the particles which are the physical basis of the simulation. However, the calculation of the Fourier transform via a Fast Fourier Transform (FFT) method is extremely fast, as it only needs of order NloqN operations, which is the advantage of this method. Here N denotes the number of mesh cells. In this approach the computational costs do not depend on the details of the particle distribution which both an advantage and a limitation depending on the problem it deals with. There are many schemes to assign the mass density to the mesh.

⁵Josh Barnes and Piet Hut. "A hierarchical O(N log N) force-calculation algorithm". In: *Nature* 324.6096 (1986), pp. 446–449. DOI: 10.1038/324446a0. URL: http://dx.doi.org/10.1038/324446a0.

The simplest method is the "Nearest-Grid-Point" (NGP). Here, each particle is assigned to the closest mesh point, and the density at each mesh point is the total mass assigned to the point divided by the cell volume. One of its drawbacks is that it gives forces that are discontinuous. The "Cloud-in-a-Cell" (CIC) method is a better approximation to the force: it distributes every particle over the nearest 8 grid cells, and then weighs their contribution to each mesh point by the overlapping volume, which is obtained by assuming the particle to have a cubic shape of the same volume as the mesh cells. The CIC method gives continuous forces, but discontinuous first derivatives of the forces (Monaghan et al 1988).⁶ A more accurate scheme is the "Triangular-Shaped-Cloud" (TSC) method (see Hockney & Eastwood 1988).⁷

In the Adaptive Mesh Refinement (AMR) techniques, larger meshes are sub divided into smaller ones wherever higher precession is needed. The Poisson equation on the refinement meshes are solved for which the boundary values are obtained by interpolating the gravitational potential from the parent grid (Dirichlet condition). In such algorithms, the boundaries of the refinement meshes can have an arbitrary shape. The Poisson equation on these meshes can be solved using the relaxation method (Hockney & Eastwood 1988;⁸ Press et al. 1992⁹), which is relatively fast and efficient in dealing with complicated boundaries. (See Figure 1.2)¹⁰



FIGURE 1.2: A slice through the refinement structure (the base grid is not shown) in a Λ CDM simulation (left panel) and the corresponding slice through the particle distribution (middle panel). The area enclosed by the square is enlarged in the right panel. Taken from Kravtsov et al. (1997).

⁶J.J. Monaghan. "An introduction to SPH". in: *Computer Physics Communications* 48.1 (1988), pp. 89 -96. ISSN: 0010-4655. DOI: http://dx.doi.org/10.1016/0010-4655(88)90026-4. URL: http://www.sciencedirect.com/science/article/pii/0010465588900264.

⁷RW Hockney and JW Eastwood. "Computer Simulation Using Particles 1988". In: *Adam Hilger* (1988), pp. 120–128.

⁸Ibid.

⁹W. H. Press et al. *Numerical recipes in C. The art of scientific computing*. 1992.

¹⁰Andrey V Kravtsov, Anatoly A Klypin, and Alexei M Khokhlov. "Adaptive refinement tree: a new high-resolution N-body code for cosmological simulations". In: *The Astrophysical Journal Supplement Series* 111.1 (1997), p. 73.

Hybrids (TreePM/P³M)

Hybrid methods can be constructed as a combination of the particle-mesh method and the tree algorithm. In TreePM methods (Bagla 2002;¹¹ Bagla & Khandai 2009¹²) the potential is divided in the Fourier space into a long-range and a short-range part:

$$\Phi = \Phi_{long} + \Phi_{short}$$

Couchman (1991) presented an improved version of the P^3M method, by proposing spatially adaptive mesh refinements in regions with high particle density (Adaptive P^3M or AP^3M).

Introducing Hydrodynamics into Simulation

The baryonic content of the Universe can typically be described as an ideal fluid. Therefore, to follow the evolution of the fluid, one usually has to solve the set of hydrodynamic equations which are the Euler equation, the Continuity equation and the First Law of thermodynamics. They are a complete set of equations provided one has an equation of state, relating the pressure P to the internal energy (per unit mass) u. A variety of numerical schemes for solving the coupled system of collisional baryonic matter and collisionless dark matter have been developed in the past decades. They fall into two categories: particle methods, which discretise mass, and grid-based methods, which discretise space.

The grid-based methods solve these equations based on Eulerian scheme. Two distict type of variables are involved. Primitive variables, which determine the thermodynamic properties, (e.g ρ , \vec{v} or P) and conservative variables which define the conservation laws, (e.g. ρ , $\rho \vec{v}$ or ρ u). These methods don't particularly perform well in high density regions and cannot accurately simulate Galaxy mergers as they don't tend to conserve angular momentum (Springel 2005).

The particle (Lagrangian) methods include variants of smoothed particle hydrodynamics (SPH; Gingold & Monaghan 1977;¹³ Lucy 1977,¹⁴ Evrard (1988),¹⁵ Hernquist

¹¹J. S. Bagla. "TreePM: A code for cosmological N-body simulations". In: *Journal of Astrophysics and Astronomy* 23.3 (2002), pp. 185–196. ISSN: 0973-7758. DOI: 10.1007/BF02702282. URL: http://dx.doi.org/10.1007/BF02702282.

¹²J. S. Bagla and Nishikanta Khandai. "The Adaptive TreePM: an adaptive resolution code for cosmological N-body simulations". In: *Monthly Notices of the Royal Astronomical Society* 396.4 (2009), p. 2211. DOI: 10.1111/j.1365-2966.2009.14880.x. eprint: /oup/backfile/content_public/journal/mnras/396/4/10.1111/j.1365-2966.2009.14880.x/3/mnras0396-2211.pdf. URL: +http://dx.doi.org/10.1111/j.1365-2966.2009.14880.x.

¹³R. A. Gingold and J. J. Monaghan. "Smoothed particle hydrodynamics - Theory and application to non-spherical stars". In: *Monthly Notices of the Royal Astronomical Society* 181 (1977), pp. 375–389. DOI: 10.1093/mnras/181.3.375.

¹⁴Leon B Lucy. "A numerical approach to the testing of the fission hypothesis". In: *The astronomical journal* 82 (1977), pp. 1013–1024.

¹⁵A. E. Evrard. "Beyond N-body - 3D cosmological gas dynamics". In: *Monthly Notices of the Royal Astronomical Society* 235 (1988), pp. 911–934. DOI: 10.1093/mnras/235.3.911.

& Katz (1989),¹⁶ Navarro & White (1993),¹⁷ Couchman et al. (1995) (Hydra),¹⁸ Steinmetz (1996a) (GRAPESPH),¹⁹ Owen et al. (1998),²⁰ Rasio (2000)²¹ and Springel (2005) (GADGET)²²). The SPH method solves the Lagrangian form of the Euler equations and can perform very well in high-density regions, but it works poorly in low- density regions. It also suffers from poor resolution in shocked regions (this is attributed to sizable artificial viscosity).

SPH introduces unphysical pressure forces on particles in regions where there are steep density gradients, in particular near contact discontinuities. This results in a boundary gap of the size comparable to SPH smoothing kernel radius, over which interactions become spurious. Nevertheless, in the cosmological context, the adaptive nature of the SPH method compensates for such shortcomings, thus making SPH the most commonly used method in numerical hydrodynamical cosmology (Dolag et al 2008).²³

Adding Complexities

Cooling: In cosmological applications, one is usually interested in structures with virial temperatures larger than 10^4 K. This calls for implementation of a cooling function, $\Lambda(u, \rho)$. For a plasma with primordial composition of H and He, the cooling processes are collisional excitation of H_I and He_{II} , collisional ionisation of H_I , He_I and He_{II} , standard recombination of H_{II} , He_{II} and He_{III} , dielectric recombination of He_{II} , and free-free emission (Bremsstrahlung) (Dolag et al 2008).²⁴ This leads to a cooling function $\Lambda(u) \propto \rho^2$.

Star formation and feedback: Including radiative losses in simulations causes numerical problems. Firstly, cooling is a runaway process and at the typical densities reached at the centres of galaxy clusters, the cooling time becomes significantly shorter than the Hubble time. As a consequence, a large fraction of the baryonic component can cool down and condense out of the hot phase in a single time step of the simulation which cannot be accurately evaluated. Secondly, since cooling is proportional to the square of the gas density, its efficiency is quite sensitive to the numerical resolution. To deal with these issues, one has to include in the code a suitable feedback model to convert the reservoir of cold and dense gas into collisionless stars. This stellar component should represent the energy feedback from supernova explosions, part of which would heat up the cold gas, so as to counteract the cooling catastrophe. As for star formation, a model was introduced by Katz

²³Dolag et al., "Simulation Techniques for Cosmological Simulations", op. cit.
 ²⁴Ibid.

¹⁶Lars Hernquist and Neal Katz. "TREESPH-A unification of SPH with the hierarchical tree method". In: *The Astrophysical Journal Supplement Series* 70 (1989), pp. 419–446.

¹⁷Simon DM White et al. "The baryon content of galaxy clusters-a challenge to cosmological orthodoxy". In: (1993).

¹⁸HMP Couchman, PA Thomas, and FR Pearce. "Hydra: An Adaptive–Mesh Implementation of PPPM–SPH". in: *arXiv preprint astro-ph/9409058* (1994).

¹⁹Matthias Steinmetz. "GRAPESPH: cosmological smoothed particle hydrodynamics simulations with the special-purpose hardware GRAPE". in: *Monthly Notices of the Royal Astronomical Society* 278.4 (1996), pp. 1005–1017.

²⁰J. Michael Owen et al. "Cosmological Simulations with Scale-Free Initial Conditions. I. Adiabatic Hydrodynamics". In: *The Astrophysical Journal* 503.1 (1998), p. 16. URL: http://stacks.iop.org/0004-637X/503/i=1/a=16.

²¹Frederic A Rasio. "Particle methods in astrophysical fluid dynamics". In: *Progress of Theoretical Physics Supplement* 138 (2000), pp. 609–621.

²²Springel, "The cosmological simulation code GADGET-2", op. cit.

et al. (1996),²⁵ which is often used in cosmological simulations. Assuming that all stars with masses larger than $8M_{\odot}$ will end as type-II supernovae (SN II), one can calculate the total amount of energy (typically 10^{51} erg per supernova) that each star particle can release to the surrounding gas. Under the approximation that the typical lifetime of massive stars which explode as SN II does not exceed the typical time step of the simulation (fractions of Hubble time), this is done in the so–called "instantaneous recycling approximation", with the feedback energy coupled isotropically in the surrounding gas in the same step.

1.2.4 Initial Conditions

Having robust and well justified initial conditions is one of the vital aspects of any numerical simulation. For cosmological purposes, observations of the large–scale distribution of galaxies and observations of the CMB agree to a good precision with the theoretical expectation that the growth of structures that start from a Gaussian random field of initial density fluctuations. Failing to setup proper initial condition can lead to spurious results even if the subsequent modelling is very well in performance.

1.2.5 Comparison between AMR and SPH codes

One of the most popular codes implementing Lagrangian Smooth Particle Hydrodynamics (SPH) is GADGET.²⁶ Its Adaptive Mesh Refinement counterpart is ENZO.²⁷ The two codes solve the same physics but implements fundamentally different algorithm for gravity solver and for solving baryon hydrodynamics. One of the most widely known code comparison work is the Santa Barbara cluster comparison project.²⁸

Gravitational effects are the primary candidate to determine structure formation in the universe. Hydrodyamics is a secondary factor in most if the large scales of structure formation. So comparing the SPH and AMR hydrodynamics require convolving them with their gravity solving algorithms as gravity often determines how hydrodynamics will function. Both ENZO and GADGET produces halos at almost same places. However, individual masses exhibit differences.²⁹ O'shea et al (2005³⁰) found that the computation of force graviational accuracy in ENZO is less than that of GADGET at initial stages of the run as adaptive refinements take a number of time to set in. However as time progresses, the ENZO accuracy increases (at smaller redshifts) due to addition of higher levels of grid. As far as the hydrodynamics is concerned, both the current versions of GADGET and ENZO has been updated to conserve entropy in adiabatic problems. The following image is taken from the code comparison work of O'shea et al (2005³¹).

²⁵N. Katz, D. H. Weinberg, and L. Hernquist. "Cosmological Simulations with TreeSPH". in: *Astrophysical Journal Supplement* 105 (July 1996), p. 19. DOI: 10.1086/192305. eprint: astro-ph/9509107.

²⁶V. Springel. "The cosmological simulation code GADGET-2". In: *Monthly Notices of the Royal Astronomical Society* 364 (Dec. 2005), pp. 1105–1134. DOI: 10.1111/j.1365-2966.2005.09655.x. eprint: astro-ph/0505010.

²⁷Greg L Bryan et al. "Enzo: An adaptive mesh refinement code for astrophysics". In: *The Astrophysical Journal Supplement Series* 211.2 (2014), p. 19.

²⁸CS Frenk et al. "The Santa Barbara cluster comparison project: a comparison of cosmological hydrodynamics solutions". In: *The Astrophysical Journal* 525.2 (1999), p. 554.

²⁹Brian W O'shea et al. "Comparing AMR and SPH cosmological simulations. I. Dark matter and adiabatic simulations". In: *The Astrophysical Journal Supplement Series* 160.1 (2005), p. 1.

³⁰Ibid.

³¹Ibid.



FIGURE 1.3: Projected dark matter (top row) and gas mass (bottom row) distribution for GADGET and Enzo in a slab of size $3 \times 3 \times 0.75(h^{-1}Mpc)^3$. For GADGET (left column), the run used with 2×64^3 particles. For ENZO (right column), the run with 64^3 dark matter particles and 128^3 root grid was used.

1.2.6 Examples of Cosmological Simulators

- ATHENA
- EAGLE
- ENZO
- FLASH
- GADGET
- GASOLINE
- OWLS
- ORION

• ROCKSTAR

Chapter 2

Environments of Supermassive Black holes

2.1 Introduction

In this chapter, I will discuss the physics of the environment around Supermassive black holes and their impact on the structure formation in the Universe. The discussion will be in relation to my work which I will elaborate in the later part of the chapter in section 2.3. I will give an account on my study of the The MassiveBlack-II Cosmological Simulation done using GADGET 2 (Springel 2005)¹ by Khandai et al 2015.²

2.2 SMBH Environments

In the vicinity of SMBH, gases have of high luminosity (energy emission corresponding to temperatures of around 10^8 K) and high density (up to 10^{-3} particles per cm^3). It is well suited to be studied by X-ray telescopes. Therefore, understanding the observational process of galaxy clusters calls for analysis of X-ray observations. The supermassive black holes observed at the centers of almost all present-day galaxies, had a profound impact on their environment. I highlight the principle of self-regulatory feedback, by which supermassive black holes grow until they release sufficient energy to strip off the gas from their host galaxy that feeds them. (Bromm & Loeb 2004^3).

Every bulged galaxy hosts a supermassive black hole (BH) at its center (Kormendy 2003⁴). These BHs are dormant or faint most of the time of their existence, but they can ocassionally flash in a short but intense burst of radiation that lasts for only a small fraction of the Hubble time. This acounts for the fact that bright quasars are much less abundant than their host galaxies.

The fact that the black holes play an important role in determining the behaviour of other bodies in their environment and this finally leads to the grand and intricate structure of our current universe can be seen from various observations. The most leading one among them is the $M - \sigma$ correlation. Here M is the mass of the black

¹Springel, "The cosmological simulation code GADGET-2", op. cit.

 $^{^{2}}$ Khandai et al., "The MassiveBlack-II simulation: the evolution of haloes and galaxies to z 0", op. cit.

³Volker Bromm and Abraham Loeb. "Formation of the First Supermassive Black Holes". In: *The Astrophysical Journal* 596.1 (2003), p. 34. URL: http://stacks.iop.org/0004-637X/596/i=1/a=34.

⁴M. Longhetti et al. "The Kormendy relation of massive elliptical galaxies at z ~ 1.5: evidence for size evolution". In: *Monthly Notices of the Royal Astronomical Society* 374 (Jan. 2007), pp. 614–626. DOI: 10.1111/j.1365-2966.2006.11171.x. eprint: astro-ph/0610241.

hole and σ is the the central stellar velocity dispersion of the galaxy. The first such relation proposed was the Faber–Jackson relation (1976⁵). A tight $M - \sigma$ correlation suggests a feedback between black hole mass and stellar velocity dispersion.⁶ So later on feedback factor was included and improved alternative models were proposed like an energy transfer model by Silk and Rees (1998⁷) and even more successful momentum transfer model by King (2003⁸).

2.2.1 Effect of SMBH on its environment

Supermassive BHs make up a small fraction, $<10^{-3}$ of the total mass in their host galaxies, and so their direct dynamical impact is limited to the central star distribution where their gravitational influence dominates. However, one may be misled to think that dynamical influence of the BH on a much larger galactic scale is insignificant. Even if the BH mass occupies a fraction as small as $\sim 10^{-4}$ of the baryonic mass in a galaxy (Bromm & Loeb 2004⁹), and only a percent of the accreted rest-mass energy leaks into the gaseous environment of the BH, this slight leakage can provide as much energy that can unbind the entire gas reservoir of the host galaxy.

The cooling time of the heated gas is typically longer than its dynamical time which is due to energy release by BH and so the gas could easily expand into the galactic halo and escape the galaxy if its initial temperature exceeds the virial temperature of the galaxy. The quasar remains active during the dynamical time of the initial gas reservoir, $\sim 10^7$ years, and fades afterwards due to the dilution of this reservoir which reduces the gas supply to the quasar (Wyithe and Loeb 2003¹⁰). Accretion is halted as soon as the quasar supplies the galactic gas enough energy to outweigh the gas binding energy. After the gas strip-off, the impact of BH on structure formation becomes really insignificant as it can now only affect environments extending a few AU (Astronomical Unit) around it. Later on, the BH growth may resume if the cold gas reservoir is replenished through a new merger.

Aside from affecting their host galaxy, quasars disturb their large-scale cosmological environment. Powerful outflows are often associated with quasars in the form of radio jets.

⁵S. M. Faber and R. E. Jackson. "Velocity dispersions and mass-to-light ratios for elliptical galaxies". In: *The Astrophysical Journal* 204 (Mar. 1976), pp. 668–683. DOI: 10.1086/154215.

⁶D. H. Gudehus. "Systematic bias in cluster galaxy data, affecting galaxy distances and evolutionary history". In: *The Astrophysical Journal* 382 (Nov. 1991), pp. 1–18. DOI: 10.1086/170687.

⁷J. Silk and M. J. Rees. "Quasars and galaxy formation". In: *Astronomy and Astrophysics* 331 (Mar. 1998), pp. L1–L4. eprint: astro-ph/9801013.

⁸A. King. "Black Holes, Galaxy Formation, and the M_{BH} - σ Relation". In: Astrophysical Journal, Letters 596 (Oct. 2003), pp. L27–L29. DOI: 10.1086/379143. eprint: astro-ph/0308342.

⁹Bromm and Loeb, "Formation of the First Supermassive Black Holes", op. cit.

¹⁰J. S. B. Wyithe and A. Loeb. "Self-regulated Growth of Supermassive Black Holes in Galaxies as the Origin of the Optical and X-Ray Luminosity Functions of Quasars". In: *The Astrophysical Journal*, 595 (Oct. 2003), pp. 614–623. DOI: 10.1086/377475. eprint: astro-ph/0304156.

2.2.2 Seeds for the growth of supermassive Black holes

Bromm & Loeb (2003)¹¹ have shown through a hydrodynamical simulation (see Figure 2.1)¹²¹³ that supermassive stars were likely to form in early galaxies at $z \sim 10$ in which the virial temperature was close to the cooling threshold of atomic hydrogen, $\sim 10^4$ K. The minimum seed BH mass from which the SMBH that we observe are formed can be identified observationally through the detection of gravitational waves from BH binaries with Advanced LIGO.¹⁴

Current observational signatures of black hole mergers¹⁵ through gravitational wave specifically the amplitude and frequency of the waves can carry the information of the initial seed mass from which the very first black holes were born. As an example of ongoing research in this field, Salcido et al (2016) used the EAGLE simulation¹⁶ results to estimate the expected event rate of gravitational wave signals from mergers of supermassive black holes and found out that mergers involving the initial seed masses can be distinguished by definite features in their gravitational waveforms providing a quantitative idea on the formation of black holes.



FIGURE 2.1: SPH simulation of the collapse of an early dwarf galaxy with a virial temperature just above the cooling threshold of atomic hydrogen and no H_2 . (Bromm and Loeb 2003). The image shows a snapshot of the gas density distribution at $z \approx 10$, indicating the formation of two compact objects near the center of the galaxy with masses of $2.2 \times 10^6 M_{\odot}$ and $3.1 \times 10^6 M_{\odot}$, respectively, and radii < 1 pc. Sub-fragmentation into lower mass clumps is inhibited as long as molecular hydrogen is dissociated by a background UV flux. These circumstances lead to the formation of supermassive stars (Loeb and Rasio 1994) that inevitably collapse and trigger the birth of supermassive black holes (Saijo et al 2002). The box size is 200 pc.

¹¹Bromm and Loeb, "Formation of the First Supermassive Black Holes", op. cit.

¹²A. Loeb and F. A. Rasio. "Collapse of primordial gas clouds and the formation of quasar black holes". In: *Astrophysical Journal* 432 (Sept. 1994), pp. 52–61. DOI: 10.1086/174548. eprint: astro-ph/9401026.

¹³Motoyuki Saijo et al. "Collapse of a Rotating Supermassive Star to a Supermassive Black Hole: Post-Newtonian Simulations". In: *The Astrophysical Journal* 569.1 (2002), p. 349. URL: http://stacks.iop.org/0004-637X/569/i=1/a=349.

¹⁴Junaid Aasi et al. "Advanced ligo". In: Classical and Quantum Gravity 32.7 (2015), p. 074001.

¹⁵Benjamin P Abbott et al. "Observation of gravitational waves from a binary black hole merger". In: *Physical review letters* 116.6 (2016), p. 061102.

¹⁶Joop Schaye et al. "The EAGLE project: simulating the evolution and assembly of galaxies and their environments". In: *Monthly Notices of the Royal Astronomical Society* 446.1 (2015), pp. 521–554.

A study of Supermassive Black hole environments using 2.3 MassiveBlack-II GADGET Simulation

The MassiveBlack-II Cosmological Simulation done using GADGET 2 (Springel 2005)¹⁷ by Khandai et al 2015.¹⁸ I have used this simulation result (snapshot) at redshift $z \approx$ 1. I have also highlighted on the aspect of how to read and process the out of the simulations and what can be the technical subtleties involved in doing so. I have elaborated on the method I followed to simulate X-ray emission from the SMBH environment. I also studied the scaling relation on black hole mass and its accretion rate and discussed some of the plausible physical causes behind the scaling relation. I compared my study with similar related work done by Chatterjee et al (2008^{19}) with SZ effect. They used the Di Matteo et al. (2008) simulation of GADGET 2 (See Appendix B).²⁰ I have also given examples of related work done to compare simulations with observations.

L_{box}	N	m_{DM}	m_{gas}	ϵ
$(h^{-1}Mpc)$	INpart	$(h^{-1}M_{\odot})$	$(h^{-1}M_{\odot})$	$(h^{-1}kpc)$
100	2×1792^3	$1.1 imes 10^7$	2.2×10^6	1.85

TABLE 2.1: The columns list the size of the simulation box, L_{box} , the number of particles (dark matter + gas) used in the simulation, N_{part} , the mass of a single dark matter particle, m_{DM} , the initial mass of a gas particle, m_{gas} , and the gravitational softening length, ϵ . All length scales are in comoving units. (Khandai et al 2014)

About The MassiveBlack-II Simulation 2.3.1

MassiveBlack-II (MBII) is high resolution hydrodynamical simulation. MBII evolves a Λ CDM cosmology in a cubical comoving volume of $V_{box} = \left(\frac{100Mpc}{h}\right)^3$ and is able to resolve halos of mass $M_{halo} = 10^9 \frac{M}{h}$. It is the highest resolution simulation of this size which includes a self-consistent model for star formation, black hole accretion and associated feedback. It was run on a large-scale computing facility with 100,000 compute cores named the Cray XT5, "Kraken".

The initial conditions for MBII were generated with the CMBFAST transfer function (Zaldarriaga and Seljak 2000²¹) at z = 159 and the simulation was evolved to z = 0. The cosmological parameters used were (WMAP7 cosmology; Komatsu et al. 2011²²): amplitude of mass fluctuations, $\sigma_8 = 0.816$, spectral index, $n_s = 0.968$, cosmological constant parameter Ω_{Λ} = 0.725, mass density parameter Ω_m = 0.275, baryon density parameter $\Omega_b = 0.046$ and h = 0.701 (Hubble's constant in units of $100 km s^{-1} Mpc - 1$).

Star Formation and Supernova feedback has been modelled in the simulation as discussed previously in Chapter 1. In MBII BHs are modelled as collisionless sink particles within newly collapsing halos, which are identified by the FOF (Friends Of

¹⁷Springel, "The cosmological simulation code GADGET-2", op. cit.

 $^{^{18}}$ Khandai et al., "The MassiveBlack-II simulation: the evolution of haloes and galaxies to z 0", op. cit

¹⁹Chatterjee et al., "Simulations of the Sunyaev–Zel'dovich effect from quasars", op. cit.

²⁰Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies",

op. cit. ²¹Matias Zaldarriaga and Uros Seljak. "CMBFAST for spatially closed universes". In: *The Astrophys-*²¹Matias Zaldarriaga and Uros Seljak. "CMBFAST for spatially closed universes". In: *The Astrophys-*

²²Eiichiro Komatsu et al. "Seven-year wilkinson microwave anisotropy probe (WMAP*) observations: cosmological interpretation". In: The Astrophysical Journal Supplement Series 192.2 (2011), p. 18.

Friends) halofinder called on the fly at regular time intervals. A seed BH of mass $M_{seed} = 5 \times 10^5 h^{-1} M_{\odot}$ is inserted into a halo with mass $M_{halo} \ge 5 \times 10^{10} h^{-1} M_{\odot}$ if it does not already contain a BH. Once seeded, BHs grow by accreting gas in its surrounding region or by merging with other BHs which may bring in fresh supply of cold gas. Gas accretion follows a Bondi-Hoyle accretion relation:²³

$$\dot{M_{BH}} = \frac{4\pi G^2 M_{BH}^2 \rho}{(c_s^2 + v_{BH}^2)^{\frac{3}{2}}}$$

where v_{BH} is the velocity of the black hole relative to the surrounding gas, ρ and c_s are the density and sound speed of the hot and cold phase of the ISM gas.

MB II allowed the accretion rate to be mildly super-Eddington but limited it to a maximum allowed value equal of $2 \times$ Eddington rate (M_{Edd}) to prevent unphysically high values, consistent with Begelman, Volonteri, & Rees (2006);²⁴ Volonteri & Rees (2006).²⁵ The BH radiates with a bolometric luminosity which is proportional to the accretion rate, $L_{bol} = \eta M_{BH}c^2$, where η is the radiative efficiency and its standard value of 0.1 (Shakura & Sunyaev 1973)²⁶ is kept throughout, and c is the speed of light. In the simulation 5% of the radiated energy couples thermally to the surrounding gas and this energy is dumped isotropically on the neighbouring gas particles that are within the BH kernel (64 nearest neighbors) and acts as a form of feedback (Springel, Di Matteo & Hernquist 2005).²⁷ The value of 5% is the only free parameter in the model and was set using galaxy merger simulations (Di Matteo, Springel, & Hernquist 2005)²⁸ to match the normalization in the observed $M_{BH} - \sigma$ relation. BHs also grow by merging once one BH comes within the kernel of another with a relative velocity below the local gas sound speed. This model for the growth of BHs has been developed by Di Matteo, Springel, & Hernquist (2005); Springel, Di Matteo,²⁹ & Hernquist (2005).³⁰ It has been implemented and studied extensively in cosmological simulations (Sijacki et al. 2007,³¹ Colberg & Di Matteo 2008; Di Matteo et al. 2008;³² Croft et al. 2009;³³ Booth & Schaye 2009;³⁴ Sijacki,

²⁶N I Shakura and Rashid Alievich Sunyaev. "Black holes in binary systems. Observational appearance." In: *Astronomy and Astrophysics* 24 (1973), pp. 337–355.

³⁰Idem, "Modelling feedback from stars and black holes in galaxy mergers", op. cit.

³¹Debora Sijacki et al. "A unified model for AGN feedback in cosmological simulations of structure formation". In: *Monthly Notices of the Royal Astronomical Society* 380.3 (2007), pp. 877–900.

²³Bondi and Hoyle, "On the mechanism of accretion by stars", op. cit.

²⁴Mitchell C Begelman, Marta Volonteri, and Martin J Rees. "Formation of supermassive black holes by direct collapse in pre-galactic haloes". In: *Monthly Notices of the Royal Astronomical Society* 370.1 (2006), pp. 289–298.

²⁵Marta Volonteri, Stephen M Merkowitz, and Jeffrey C Livas. "Supermassive black hole mergers and cosmological structure formation". In: *AIP Conference Proceedings*. Vol. 873. 1. AIP. 2006, pp. 61–69.

²⁷ Volker Springel, Tiziana Di Matteo, and Lars Hernquist. "Modelling feedback from stars and black holes in galaxy mergers". In: *Monthly Notices of the Royal Astronomical Society* 361.3 (2005), pp. 776–794.
²⁸ Ibid.

²⁹Volker Springel, Tiziana Di Matteo, and Lars Hernquist. "Black holes in galaxy mergers: the formation of red elliptical galaxies". In: *The Astrophysical Journal Letters* 620.2 (2005), p. L79.

³²Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies", op. cit.

³³Greg L Bryan and Marie E Machacek. "The b Distribution of the Ly α Forest: Probing Cosmology and the Intergalactic Medium". In: *The Astrophysical Journal* 534.1 (2000), p. 57.

³⁴CM Booth and Joop Schaye. "Cosmological simulations of the growth of supermassive black holes and feedback from active galactic nuclei: method and tests". In: *Monthly Notices of the Royal Astronomical Society* 398.1 (2009), pp. 53–74.

Springel, & Haehnelt 2009;³⁵ Chatterjee et al. 2009³⁶), successfully reproducing basic properties of BH growth, the observed $M_{BH} - \sigma$ relation and the BH mass function (Di Matteo et al. 2008³⁷).

2.3.2 Working with MB II output at $z\approx 1$

The size of the entire MB II output file for each redshift is around 1 TB which is divided into 1024 smaller files each of the size of around 1 GB. The output is called a 'Snapshot'. It is in the form of a Fortran Unformatted Binary. The data hierarchy of this typical binary file is that each snapshot contains a header (256 bytes) which has some general informations about the file and the simulation. This is followed by data blocks. Each data block is bracketed by a 4 byte tag at the beginning and at the end. Each of these tag contains the size in bytes of the data block it is bracketing. This provide a means to quicly skip through the file to the data one is interested in. For reading the Binary file I wrote codes in Python 3 using numpy and modified an already existing python module pyGadgetReader³⁸(It couldn't be used in my work in its original form). Reading from such a large datafile can be extremely time consuming if done serially (took ~ 6 hours). So it is an efficient idea is to read the snapshot file parallely in a Beowulf Cluster setup using a LAN switch (I used 2 nodes and followed the work done by Kiepert (2013)³⁹ on developing a RaspBerrry Pi Cluster). I ran the code in Python 3 in a simple embarassingly parallel manner using MPI⁴⁰ (Sterling and Bell; Beowulf Cluster Computing with Linux).⁴¹

2.3.3 Simulation of X-ray Map around BlackHoles

X-ray astronomy greatly involves detecting clusters via the hot X-ray gas present in the ICM. Clusters are the largest objects in the universe in thermal equilibrium with masses between 10^{14} to $10^{15} M_{\odot}$. The total gas fraction in clusters is about 16% with about 13% in the ICM and 3% in galaxies. The rest of the mass consists of dark matter. The gas densities at the center of galaxy clusters could be as high as $10^{-1}cm^{-3}$ to $10^{-3}cm^{-3}$, which is different from the cosmic baryon density of $10^{-8}cm^{-3}$. The virial radius of a cluster is defined as the radius within which the mean density of the cluster is 200 times the critical density ($9.4 \times 10^{-30} gcm^{-3}$) of the universe. The gas of the cluster is heated by gravitational infall to temperatures between 1-15 keV. The total X-ray luminosity in galaxy clusters range from $10^{43} erg/s$ to $10^{46} erg/s$. (Chatterje et al 2009).

To simulate such an environment, from a gas distribution obtained from MBII output, I followed the work of Di Matteo et al. 2008.⁴² To obtain an X-ray image of the modelled cluster, I chose a projected direction and integrate over all the emission of the elements along the line of sight for each pixel in the image. The emission

³⁶Chatterjee et al., "Simulations of the Sunyaev-Zel'dovich effect from quasars", op. cit.

³⁵Colin DeGraf et al. "Growth of early supermassive black holes and the high-redshift Eddington ratio distribution". In: *The Astrophysical Journal Letters* 755.1 (2012), p. L8.

³⁷Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies", op. cit.

³⁸R. Thompson. *pyGadgetReader: GADGET snapshot reader for python*. Astrophysics Source Code Library. Nov. 2014. ascl: 1411.001.

³⁹Joshua Kiepert. "Creating a raspberry pi-based beowulf cluster". In: ().

 ⁴⁰Message P Forum. MPI: A Message-Passing Interface Standard. Tech. rep. Knoxville, TN, USA, 1994.
 ⁴¹Thomas Lawrence Sterling. Beowulf cluster computing with Linux. 2002.

⁴²Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies", op. cit.

is modelled as Bremsstrahlung which goes as $n_e^2 T^{\frac{1}{2}}$ where n_e is the free electron abundance in the SMBH environment. I have used a common B_2 Spline Smoothing Kernel for my map which is:

$$W(x,h) = \frac{\sigma}{h^{\nu}} \begin{cases} 1 - 6\left(\frac{x}{h}\right)^2 + 6\left(\frac{x}{h}\right)^3 & 0 \le \frac{x}{h} < 0.5\\ 2\left(1 - \frac{x}{h}\right)^3 & 0.5 \le \frac{x}{h} < 1\\ 0 & 1 \le \frac{x}{h} \end{cases}$$

where ν is the dimensionality (e.g. 1, 2 or 3) and σ is the normalisation

$$\sigma = \begin{cases} \frac{16}{3} & \nu = 1\\ \frac{80}{7\pi} & \nu = 2\\ \frac{8}{\pi} & \nu = 3 \end{cases}$$

This is the same smoothing kernel used in GADGET (Dolag et al 2008^{43}).

The following are the simulated X-ray maps around one of the most massive Blackholes in the MB II simulation taken at different projections (line of sight). Obviously the results of this type of simulations has a directly observable counter part unlike the Cosmological simulations where three dimensional distribution of particles are not direct observables.

⁴³Dolag et al., "Simulation Techniques for Cosmological Simulations", op. cit.



FIGURE 2.2: MB II Simulation of Khandai et al 2014 with M_{BH} ~ $10^8 M_{\odot}$. This Supermassive black hole is at the centre of the map. The x and the y axes are distances in kpc. I have considered X-ray emission from the gas particles around the black hole along an arbitrary line of sight. In this image it is the z axis (as defined in the simulation). The model of X-ray emission considered is that of Thermal Bremsstrahlung. The contribution of each particle in X-ray emission can be thought of as being washed out over a region according to a Smoothing kernel. Each pixel in the map is filled with a value corresponding to the intensity of X-ray falling on that pixel. This is found by integrating contributions from all particles at each pixel (weighed by their smoothing kernel) along the line of sight.





2.3.4 Scaling Relations

I have found out Scaling relations of black hole luminosity and mass. It shows departure from the Bondi Scaling relation⁴⁴ about which I have already discussed. I have only considered black holes with mass greater than $10^6 M_{\odot}$ and luminosity greater than $10^{38} erg/s$. I have binned the black hole population and took the average luminosity in each bin as the luminosity of that bin. Weighted standard deviations are taken as error bars and a Spearmann Correlation is studied. Following are my results:



FIGURE 2.4: Mass-Luminosity Scatter Plot using MB II simulation of Khandai et al 2014. I have considered all the black holes at redshift z = 1 in the MB-II simulation that has a luminosity above $10^{38} erg/s$ and a mass above $10^{6} M_{\odot}$. One can see extremely dense clustering of the points which can indicate that luminosity has a pretty strong correlation with black hole mass.

⁴⁴Bondi and Hoyle, "On the mechanism of accretion by stars", op. cit.



FIGURE 2.5: This is the plot of luminosity population distribution of SMBH in MB-II simulation of Khandai et al 2014. The cut-off luminosity and mass are chosen as before. This histogram has been made by choosing equally spaced luminosity ranges as bins and counting the number of black holes that go in each of the bin. This shows that black holes with luminosity of around $10^{42} erg/s$ are most abundant; a fact that can be directly verified through observations. So a scaling relation can provide insight in explaining why this is so.



FIGURE 2.6: Mass-Luminosity scaling relation of SMBH using MB II simulation of Khandai et al 2014. This scaling relation has been made by dividing the entire mass range of the black holes into several bins and taking the median mass of each bin as the mass of the black hole that is to be plotted along the x axis. The mean luminosity of all the black holes within each mass bin is assingned the luminosity that is to be plotted along the y axis. The standard deviation in luminosity of all the black holes within each mass bin normalized by \sqrt{N} is taken as the error bar. Here, N denotes the total number of black holes in a particular mass bin. As one can easily spot that the scaling relation. This result has a good correspondance to the scaling relation found

by Chatterjee et al (2008) with the Sunyaev-Zeldovich effect.



FIGURE 2.7: Mass-Luminosity Spearmann Rank distribution of SMBH using MB-II simulation of Khandai et al 2014. As I have already discussed I have divided the mass of the black holes into several bins and took the mean luminosity of all the black holes in each mass bin as the luminosity and the standard deviation in the luminosity as error bars. Now to get a Spearmann Rank distribution, I generated 1000 mock luminosity-mass data points by making a random sampling of mass and luminosity from each bin. I have considered that both mass and luminosity follows a Gaussian distribution with half of mass bin-width as the standard deviation in mass and luminosity error as standard deviation in luminosity while making the random sampling. Next I found out the Spearmann Rank for all the 1000 datasets followed by making a population distribution histogram of them. The Spearmann Rank shows very tight correlation in black hole mass and luminosity.



FIGURE 2.8: Spearmann p value for Mass-Luminosity relation of SMBH using MB II simulation of Khandai et al 2014. While calculating Spearmann rank, it becomes eesntial to calculate a p value. Here I have used the same 1000 mock datasets which I discussed in the previous image caption. In p value calculations, one has to consider two hypothesis operating. One is that the two variables are correlated and the alternative hypothesis is that they are not. Now p value is the conditional probability that the alternative hypothesis is operating given the observed dataset. So smaller p value indicates stronger correlation. In ths case, the p value is extremely small with almost no spread at all.

So summing up, these results indicate a strong correlation between the two quantities (black hole mass and luminosity). Similar results has been found by Chatterjee el at (2008)⁴⁵ at same redshift while studying the SZ effect using Di Matteo et al. (2008) simulation (See Appendix B).⁴⁶ This shows consistency in results obtained from two entirely different physical phenomena.

23

⁴⁵Chatterjee et al., "Simulations of the Sunyaev–Zel'dovich effect from quasars", op. cit.

⁴⁶Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies", op. cit.

z	$\dot{M_{BH}}$
3.0	$\log(\dot{M_{BH}}) = 0.74 \log\left(\frac{M_{BH}}{M_{\odot}}\right) - 8.1$
2.0	$\log(\dot{M_{BH}}) = 0.65 \log\left(\frac{M_{BH}}{M_{\odot}}\right) - 8.4$
1.0	$\log(\dot{M_{BH}}) = 1.4 \log\left(\frac{M_{BH}}{M_{\odot}}\right) - 15$

TABLE 2.2: Mass-Luminosity Mass-Luminosity Scaling Relation by making simulations of SZ effect. Taken from Chatterjee et al 2008.

2.4 Conclusion

In this section I will wrap up all the discussions I have done so far and elaborate on my future plans related to this project.

There is a gap between simulations and observations. Simulations generate physical properties in 3D space like particle positions, densities, etc. On the other what that is observed are 2D maps of emissions from these physical regions. So to bridge up this gap, one requires building pipelines for comparing simulations with observations. And there comes the concept of virtual telescopes and virtual observers.⁴⁷

However, comparing observations with simulations requires careful study of instrumental effects like resolution, noise and sensitivity of the instruments. For example, the effective collecting area of a telescope is often a function of the incident light frequency and this means that comparing simulations with observations at different wavelength requires modelling this dependence of telescope collecting area.

Future Plans

- The departure of the Scaling relation from Bondi Scaling relation is very interesting. I want to investigate whether Blackhole mergers cause this deviation. This requires finding out scaling relations as a function of Blackhole merger history. So I will need to study at different redshifts.
- I wish to compare my simulated X-ray maps with real X-ray observations made by Chandra by convolving my simulation with Chandra PSF.⁴⁸
- Finally I also want to improve the computing power by increasing node numbers and implementing distributed file systems like Hadoop HDFS⁴⁹ or Apache Spark⁵⁰ and possibly with GPU computing.⁵¹
- I end here by outlining the results of some the work already done in this field of Virtual Observers and Virtual Telescopes.⁵²⁵³

⁴⁷Dolag et al., "Simulation Techniques for Cosmological Simulations", op. cit.

⁴⁸Christopher Allen, Diab H Jerius, and Terrance J Gaetz. "Parameterization of the Chandra point spread function". In: *Optical Science and Technology, SPIE's 48th Annual Meeting*. International Society for Optics and Photonics. 2004, pp. 423–432.

⁴⁹Konstantin Shvachko et al. "The hadoop distributed file system". In: *Mass storage systems and technologies (MSST), 2010 IEEE 26th symposium on.* IEEE. 2010, pp. 1–10.

⁵⁰Matei Zaharia et al. "Apache Spark: a unified engine for big data processing". In: *Communications of the ACM* 59.11 (2016), pp. 56–65.

⁵¹John Nickolls et al. "Scalable parallel programming with CUDA". in: Queue 6.2 (2008), pp. 40–53.

⁵²Elena Rasia et al. "Systematics in the X-ray cluster mass estimators". In: *Monthly Notices of the Royal Astronomical Society* 369.4 (2006), pp. 2013–2024.

⁵³A. Bonaldi et al. "Sunyaev-Zel'dovich profiles and scaling relations: modelling effects and observational biases". In: *Monthly Notices of the Royal Astronomical Society* 378 (July 2007), pp. 1248–1258. DOI: 10.1111/j.1365-2966.2007.11893.x. arXiv: 0704.2535.



FIGURE 2.9: Simulated photon images in the 0.7-2 keV energy band of a simulated galaxy cluster using XMAS-2. The images are binned to 3.2 arcsec. They include background, vignetting effects, out-oftime events and the telescope optical paths. From top left to bottom right are simulations for the MOS1, PN, and MOS2 instruments on board of the XMM-Newton satellite and for the ACIS-S3 instrument onboard of the Chandra satellite.Taken from Elena Rasia, see Rasia et al. (2006).



FIGURE 2.10: Maps for the SZ decrement for a simulated galaxy cluster. The original map extracted from the hydrodynamic simulation, and the same map in the simulated observation (t = 34 hour) which assumes the AMI interferometric response, are shown in the left and right panel, respectively. The side of each map corresponds to 16 arcmin. The colour scale is shown at the bottom of each panel. Taken from Bonaldi et al. (2007).

Appendix A

Cosmological Parameters

A.1 Planck Collaboration Cosmological parameters 2015

Parameter	TT+lowP+lensing 68% limits	TT,TE,EE+lowP+lensing+ext 68% limits
$\overline{n_{\rm s}}$	0.9677 ± 0.0060	0.9667 ± 0.0040
H_0	67.81 ± 0.92	67.74 ± 0.46
Ω_{Λ}	0.692 ± 0.012	0.6911 ± 0.0062
$\Omega_m \ldots \ldots \ldots \ldots \ldots \ldots$	0.308 ± 0.012	0.3089 ± 0.0062
$\Omega_{ m b}h^2$	0.02226 ± 0.00023	0.02230 ± 0.00014
$\Omega_{ m c}h^2$	0.1186 ± 0.0020	0.1188 ± 0.0010
σ_8	0.8149 ± 0.0093	0.8159 ± 0.0086
<i>Z</i> _{re}	$8.8^{+1.7}_{-1.4}$	$8.8^{+1.2}_{-1.1}$
Age/Gyr	13.799 ± 0.038	13.799 ± 0.021

FIGURE A.1: From the Planck 2015 results. XIII.

Appendix **B**

Di Matteo et al. 2008 GADGET 2 Cosmological Simulation

B.1 Parameters of the Di Matteo et al. 2008 simulation

Taken from Di Matteo et al 2008¹

Run	Box Size $(h^{-1}Mpc)$	N_P	$m_{DM} (h^{-1} M_{\odot})$	m_{gas} ($h^{-1}M_{\odot}$)	$\epsilon (h^{-1}kpc)$	z_{end}
D4	33.75	2×216^3	2.75×10^8	4.24×10^7	6.25	0
D6 (BH Cosmo)	33.75	2×486^3	2.75×10^7	4.24×10^6	2.73	1.0

TABLE B.1: Parameters of the Di Matteo et al. 2008 simulation.

¹Di Matteo et al., "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies", op. cit.

Bibliography

- Aasi, Junaid et al. "Advanced ligo". In: *Classical and Quantum Gravity* 32.7 (2015), p. 074001.
- Abbott, Benjamin P et al. "Observation of gravitational waves from a binary black hole merger". In: *Physical review letters* 116.6 (2016), p. 061102.
- Allen, Christopher, Diab H Jerius, and Terrance J Gaetz. "Parameterization of the Chandra point spread function". In: Optical Science and Technology, SPIE's 48th Annual Meeting. International Society for Optics and Photonics. 2004, pp. 423– 432.
- Bagla, J. S. "TreePM: A code for cosmological N-body simulations". In: *Journal of Astrophysics and Astronomy* 23.3 (2002), pp. 185–196. ISSN: 0973-7758. DOI: 10.1007/BF02702282. URL: http://dx.doi.org/10.1007/BF02702282.
- Bagla, J. S. and Nishikanta Khandai. "The Adaptive TreePM: an adaptive resolution code for cosmological N-body simulations". In: *Monthly Notices of the Royal Astronomical Society* 396.4 (2009), p. 2211. DOI: 10.1111/j.1365-2966.2009. 14880.x. eprint: /oup/backfile/content_public/journal/mnras/ 396/4/10.1111/j.1365-2966.2009.14880.x/3/mnras0396-2211. pdf. URL: +http://dx.doi.org/10.1111/j.1365-2966.2009.14880. x.
- Barnes, Josh and Piet Hut. "A hierarchical O(N log N) force-calculation algorithm". In: *Nature* 324.6096 (1986), pp. 446–449. DOI: 10.1038/324446a0. URL: http: //dx.doi.org/10.1038/324446a0.
- Begelman, Mitchell C, Marta Volonteri, and Martin J Rees. "Formation of supermassive black holes by direct collapse in pre-galactic haloes". In: *Monthly Notices of the Royal Astronomical Society* 370.1 (2006), pp. 289–298.
- Bonaldi, A. et al. "Sunyaev-Zel'dovich profiles and scaling relations: modelling effects and observational biases". In: *Monthly Notices of the Royal Astronomical Society* 378 (July 2007), pp. 1248–1258. DOI: 10.1111/j.1365-2966.2007.11893.x. arXiv: 0704.2535.
- Bondi, Hermann and Fred Hoyle. "On the mechanism of accretion by stars". In: *Monthly Notices of the Royal Astronomical Society* 104.5 (1944), pp. 273–282.
- Bondi, HJ. "On spherically symmetrical accretion". In: *Monthly Notices of the Royal Astronomical Society* 112.2 (1952), pp. 195–204.
- Booth, CM and Joop Schaye. "Cosmological simulations of the growth of supermassive black holes and feedback from active galactic nuclei: method and tests". In: *Monthly Notices of the Royal Astronomical Society* 398.1 (2009), pp. 53–74.
- Bromm, Volker and Abraham Loeb. "Formation of the First Supermassive Black Holes". In: *The Astrophysical Journal* 596.1 (2003), p. 34. URL: http://stacks. iop.org/0004-637X/596/i=1/a=34.
- Bryan, Greg L and Marie E Machacek. "The b Distribution of the Lyα Forest: Probing Cosmology and the Intergalactic Medium". In: *The Astrophysical Journal* 534.1 (2000), p. 57.
- Bryan, Greg L et al. "Enzo: An adaptive mesh refinement code for astrophysics". In: *The Astrophysical Journal Supplement Series* 211.2 (2014), p. 19.

- Chatterjee, Suchetana et al. "Simulations of the Sunyaev–Zel'dovich effect from quasars". In: *Monthly Notices of the Royal Astronomical Society* 390.2 (2008), pp. 535–544.
- Couchman, HMP, PA Thomas, and FR Pearce. "Hydra: An Adaptive–Mesh Implementation of PPPM–SPH". In: *arXiv preprint astro-ph/9409058* (1994).
- DeGraf, Colin et al. "Growth of early supermassive black holes and the high-redshift Eddington ratio distribution". In: *The Astrophysical Journal Letters* 755.1 (2012), p. L8.
- Di Matteo, T. et al. "Direct Cosmological Simulations of the Growth of Black Holes and Galaxies". In: *The Astrophysical Journal* 676, 33-53 (Mar. 2008), pp. 33–53. DOI: 10.1086/524921. arXiv: 0705.2269.
- Dolag, K. et al. "Simulation Techniques for Cosmological Simulations". In: *Space Science Reviews* 134.1 (2008), pp. 229–268. ISSN: 1572-9672. DOI: 10.1007/s11214-008-9316-5. URL: http://dx.doi.org/10.1007/s11214-008-9316-5.
- Evrard, A. E. "Beyond N-body 3D cosmological gas dynamics". In: *Monthly Notices* of the Royal Astronomical Society 235 (1988), pp. 911–934. DOI: 10.1093/mnras/ 235.3.911.
- Faber, S. M. and R. E. Jackson. "Velocity dispersions and mass-to-light ratios for elliptical galaxies". In: *The Astrophysical Journal* 204 (Mar. 1976), pp. 668–683. DOI: 10.1086/154215.
- Forum, Message P. MPI: A Message-Passing Interface Standard. Tech. rep. Knoxville, TN, USA, 1994.
- Frenk, CS et al. "The Santa Barbara cluster comparison project: a comparison of cosmological hydrodynamics solutions". In: *The Astrophysical Journal* 525.2 (1999), p. 554.
- Gingold, R. A. and J. J. Monaghan. "Smoothed particle hydrodynamics Theory and application to non-spherical stars". In: *Monthly Notices of the Royal Astronomical Society* 181 (1977), pp. 375–389. DOI: 10.1093/mnras/181.3.375.
- Gudehus, D. H. "Systematic bias in cluster galaxy data, affecting galaxy distances and evolutionary history". In: *The Astrophysical Journal* 382 (Nov. 1991), pp. 1–18. DOI: 10.1086/170687.
- Hernquist, Lars and Neal Katz. "TREESPH-A unification of SPH with the hierarchical tree method". In: *The Astrophysical Journal Supplement Series* 70 (1989), pp. 419– 446.
- Hockney, RW and JW Eastwood. "Computer Simulation Using Particles 1988". In: *Adam Hilger* (1988), pp. 120–128.
- Hoyle, F. and R. A. Lyttleton. "The effect of interstellar matter on climatic variation". In: *Proceedings of the Cambridge Philosophical Society* 35 (1939), p. 405. DOI: 10.1017/S0305004100021150.
- Katz, N., D. H. Weinberg, and L. Hernquist. "Cosmological Simulations with TreeSPH". In: *Astrophysical Journal Supplement* 105 (July 1996), p. 19. DOI: 10.1086/192305. eprint: astro-ph/9509107.
- Khandai, Nishikanta et al. "The MassiveBlack-II simulation: the evolution of haloes and galaxies to z 0". In: *Monthly Notices of the Royal Astronomical Society* 450.2 (2015), pp. 1349–1374.
- Kiepert, Joshua. "Creating a raspberry pi-based beowulf cluster". In: ().
- King, A. "Black Holes, Galaxy Formation, and the M_{BH}-σ Relation". In: Astrophysical Journal, Letters 596 (Oct. 2003), pp. L27–L29. DOI: 10.1086/379143. eprint: astro-ph/0308342.
- Komatsu, Eiichiro et al. "Seven-year wilkinson microwave anisotropy probe (WMAP*) observations: cosmological interpretation". In: *The Astrophysical Journal Supplement Series* 192.2 (2011), p. 18.

- Kravtsov, Andrey V, Anatoly A Klypin, and Alexei M Khokhlov. "Adaptive refinement tree: a new high-resolution N-body code for cosmological simulations". In: *The Astrophysical Journal Supplement Series* 111.1 (1997), p. 73.
- Loeb, A. and F. A. Rasio. "Collapse of primordial gas clouds and the formation of quasar black holes". In: *Astrophysical Journal* 432 (Sept. 1994), pp. 52–61. DOI: 10. 1086/174548. eprint: astro-ph/9401026.
- Longhetti, M. et al. "The Kormendy relation of massive elliptical galaxies at z ~ 1.5: evidence for size evolution". In: *Monthly Notices of the Royal Astronomical Society* 374 (Jan. 2007), pp. 614–626. DOI: 10.1111/j.1365–2966.2006.11171.x. eprint: astro-ph/0610241.
- Lucy, Leon B. "A numerical approach to the testing of the fission hypothesis". In: *The astronomical journal* 82 (1977), pp. 1013–1024.
- Monaghan, J.J. "An introduction to SPH". In: Computer Physics Communications 48.1
 (1988), pp. 89 -96. ISSN: 0010-4655. DOI: http://dx.doi.org/10.1016/
 0010 4655(88) 90026 4. URL: http://www.sciencedirect.com/
 science/article/pii/0010465588900264.
- Nickolls, John et al. "Scalable parallel programming with CUDA". In: *Queue* 6.2 (2008), pp. 40–53.
- Owen, J. Michael et al. "Cosmological Simulations with Scale-Free Initial Conditions. I. Adiabatic Hydrodynamics". In: *The Astrophysical Journal* 503.1 (1998), p. 16. URL: http://stacks.iop.org/0004-637X/503/i=1/a=16.
- O'shea, Brian W et al. "Comparing AMR and SPH cosmological simulations. I. Dark matter and adiabatic simulations". In: *The Astrophysical Journal Supplement Series* 160.1 (2005), p. 1.
- Press, W. H. et al. Numerical recipes in C. The art of scientific computing. 1992.
- Rasia, Elena et al. "Systematics in the X-ray cluster mass estimators". In: *Monthly Notices of the Royal Astronomical Society* 369.4 (2006), pp. 2013–2024.
- Rasio, Frederic A. "Particle methods in astrophysical fluid dynamics". In: *Progress of Theoretical Physics Supplement* 138 (2000), pp. 609–621.
- Ryden, Barbara Sue. Introduction to cosmology. Includes bibliographical references (pages [235]-237) and index. San Francisco: Addison-Wesley, [2003] ©2003, [2003]. URL: https://search.library.wisc.edu/catalog/999940494802121.
- Saijo, Motoyuki et al. "Collapse of a Rotating Supermassive Star to a Supermassive Black Hole: Post-Newtonian Simulations". In: *The Astrophysical Journal* 569.1 (2002), p. 349. URL: http://stacks.iop.org/0004-637X/569/i=1/a=349.
- Schaye, Joop et al. "The EAGLE project: simulating the evolution and assembly of galaxies and their environments". In: *Monthly Notices of the Royal Astronomical Society* 446.1 (2015), pp. 521–554.
- Shakura, N I and Rashid Alievich Sunyaev. "Black holes in binary systems. Observational appearance." In: *Astronomy and Astrophysics* 24 (1973), pp. 337–355.
- Shvachko, Konstantin et al. "The hadoop distributed file system". In: *Mass storage systems and technologies (MSST), 2010 IEEE 26th symposium on*. IEEE. 2010, pp. 1–10.
- Sijacki, Debora et al. "A unified model for AGN feedback in cosmological simulations of structure formation". In: *Monthly Notices of the Royal Astronomical Society* 380.3 (2007), pp. 877–900.
- Silk, J. and M. J. Rees. "Quasars and galaxy formation". In: *Astronomy and Astro-physics* 331 (Mar. 1998), pp. L1–L4. eprint: astro-ph/9801013.

- Springel, V. "The cosmological simulation code GADGET-2". In: *Monthly Notices of the Royal Astronomical Society* 364 (Dec. 2005), pp. 1105–1134. DOI: 10.1111/j. 1365–2966.2005.09655.x. eprint: astro-ph/0505010.
- Springel, Volker. "The cosmological simulation code GADGET-2". In: *Monthly notices of the royal astronomical society* 364.4 (2005), pp. 1105–1134.
- Springel, Volker, Tiziana Di Matteo, and Lars Hernquist. "Black holes in galaxy mergers: the formation of red elliptical galaxies". In: *The Astrophysical Journal Letters* 620.2 (2005), p. L79.
- "Modelling feedback from stars and black holes in galaxy mergers". In: *Monthly* Notices of the Royal Astronomical Society 361.3 (2005), pp. 776–794.
- Steinmetz, Matthias. "GRAPESPH: cosmological smoothed particle hydrodynamics simulations with the special-purpose hardware GRAPE". In: *Monthly Notices of the Royal Astronomical Society* 278.4 (1996), pp. 1005–1017.
- Sterling, Thomas Lawrence. Beowulf cluster computing with Linux. 2002.
- Sunyaev, R. A. and Y. B. Zeldovich. "The Observations of Relic Radiation as a Test of the Nature of X-Ray Radiation from the Clusters of Galaxies". In: *Comments on Astrophysics and Space Physics* 4 (Nov. 1972), p. 173.
- Thompson, R. *pyGadgetReader: GADGET snapshot reader for python*. Astrophysics Source Code Library. Nov. 2014. ascl: 1411.001.
- Vaucouleurs, G. de. "The Large-Scale Distribution of Galaxies and Clusters of Galaxies". In: Publications of the Astronomical Society of the Pacific 83.492 (1971), p. 113. URL: http://stacks.iop.org/1538-3873/83/i=492/a=113.
- Volonteri, Marta, Stephen M Merkowitz, and Jeffrey C Livas. "Supermassive black hole mergers and cosmological structure formation". In: *AIP Conference Proceedings*. Vol. 873. 1. AIP. 2006, pp. 61–69.
- White, Simon DM, CS Frenk, and Marc Davis. "Clustering in a neutrino-dominated universe". In: *The Astrophysical Journal* 274 (1983), pp. L1–L5.
- White, Simon DM et al. "The baryon content of galaxy clusters-a challenge to cosmological orthodoxy". In: (1993).
- Wyithe, J. S. B. and A. Loeb. "Self-regulated Growth of Supermassive Black Holes in Galaxies as the Origin of the Optical and X-Ray Luminosity Functions of Quasars". In: *The Astrophysical Journal*, 595 (Oct. 2003), pp. 614–623. DOI: 10. 1086/377475. eprint: astro-ph/0304156.
- Zaharia, Matei et al. "Apache Spark: a unified engine for big data processing". In: *Communications of the ACM* 59.11 (2016), pp. 56–65.
- Zaldarriaga, Matias and Uros Seljak. "CMBFAST for spatially closed universes". In: *The Astrophysical Journal Supplement Series* 129.2 (2000), p. 431.